COMPARATIVE ANALYSIS OF RAIL TRAFFIC ROUTE ASSIGNMENT: EVIDENCE FROM THE TOKYO METROPOLITAN AREA

by

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Abstract

This paper empirically compares the performance of six traffic assignment methods using the same empirical dataset of route choice. Multinomial logit (MNL), structured multinomial probit (SMNP), user equilibrium (UE), logit-based stochastic user equilibrium (SUE), probit-based SUE, and all-or-nothing (AON) assignment are applied to the comparative analysis. The four methods of generating the route choice set are also compared for the stochastic traffic assignment methods. The revealed preference data of urban rail route choice in the Tokyo Metropolitan Area are used for the case analysis. The empirical case analysis shows that the accuracy of the probit-based SUE is the best. However, the probit-based SUE requires the longest computation time, which may be too long from the practical viewpoint. It also shows that the heuristics to generate the choice set influence the accuracy, while incorporating the route commonality and in-vehicle congestion significantly improves the accuracy.

Keywords: Urban rail demand, network assignment, Tokyo Metropolitan Area, comparative analysis
1.0 INTRODUCTION

Traffic forecasts are routinely used to dimension the construction of public transit infrastructure projects. Here, accuracy in forecasts is a point of considerable importance for the effective allocation of scarce funds (Flyvbjerg 2006). However, the inaccuracy of travel demand forecasts in past public transit projects has been criticized by many researchers (Kain 1990; Ryan 2004; Zhao and Kockelman 2002). Flynberg (2006) points out that the patronage estimates used by planners of rail infrastructure development are highly, systematically, and significantly misleading. The risks generated by misleading forecasts are typically ignored or downplayed in public transit infrastructure planning, to the detriment of social and economic welfare. This paper empirically analyzes the extent to which the choice of travel demand analysis methods impacts the estimated results of travel demand in the public transit system.

In general, as Ortuzar and Willumsen (1990) show, there are several features of transportation models that must be taken into account when specifying an analytical approach: the decision-making context, accuracy required, availability of suitable data, state-of-the-art modeling, resources available for the study, data processing requirements, and levels of training and skills of the analysts. However, in practice, the choice of method often depends on the analyst’s preference or his/her knowledge and experience. This may be because training costs are usually high and it is sometimes better to use an existing model that is well understood than to embark on acquiring and learning the use of an advanced one. However, even if the same data set is used, the estimated results may vary among the travel demand forecast methods. This could render the analysis inadequate and/or negatively affect decision making.

Some studies have compared performance among different travel demand models. For example, Kuwahara (1988) empirically applies the following four
assignment methods to the road network in the Tokyo Metropolitan Area: incremental assignment, incremental assignment incorporating stochastic choice, user equilibrium (UE), and stochastic user equilibrium (SUE). On the basis of a comparison among the four methods, he shows that the fitness of estimated traffic flows is higher in incremental assignment incorporating the stochastic choice and SUE than in the other methods. Lam and Lo (2004) also applies the following four assignment methods to a simplified road network in Hong Kong: UE, probit-based SUE, C-logit-based SUE, and logit-based SUE. They conclude that the probit-based SUE has the highest fitness, while UE has the lowest. Although these studies empirically analyze the characteristics of assignment methods in road networks, no study has analyzed the different assignment methods in rail networks to the best of our knowledge.

This paper extends our previous work (Kato et al. 2007), which empirically analyzed the characteristics of the traffic assignment methods in the urban rail route demand forecast in the Tokyo Metropolitan Area. The following six methods are analyzed: multinomial logit (MNL) model, structured multinomial probit (SMNP) model, UE, logit-based SUE, probit-based SUE, and all-or-nothing (AON) assignment. Additionally, the four heuristic methods to generate the route choice set are also included in our analysis. We will compare them empirically with the same dataset. This paper is structured as follows. Section 2 provides a brief historical overview of urban rail demand forecast in Tokyo. Section 3 presents the study approach and the methods compared in the study. Next, Section 4 shows the data used in the empirical analysis. Section 5 presents the results of empirical comparisons among the six methods. Finally, Section 6 summarizes the paper and points out further issues.
2.0 URBAN RAIL DEMAND FORECAST IN THE TOKYO METROPOLITAN AREA

On June 12, 1872, two daily train services started operating between Shinagawa and Yokohama, marking the launch of regular passenger train services in Japan (Aoki 1994). This was also the first rail service in the Tokyo Metropolitan Area. It was in 1920 that the first master plan for developing subways lines with a total length of 80 km was adopted as part of the Tokyo Metropolitan City Plan (Kato 1996). A number of urban railway plans had been proposed after World War II. See works such as Aoki et al. (2000) and Morich et al. (2001) for a detailed history of urban rail planning in Tokyo.

It was in 1972 that a mathematical travel demand analysis was first introduced into urban rail planning in Tokyo. In 1985, a four-step travel demand model was introduced into rail demand analysis. MNL models were used for the modal choice and rail route choice models. The latest urban rail development plan in the Tokyo Metropolitan Area was proposed by the Transport Policy Committee and commissioned by the Minister of Transport, Japan, in 2000. The four-step travel demand model was again used for travel demand forecasts. The MNL model was used for the modal choice analysis, while a probit-based SUE method was used for the route choice analysis. The probit model is used because it is necessary to incorporate the commonality of routes into the rail route choice analysis. A huge urban rail network with high density has been already developed in the Tokyo Metropolitan Area. Thus, to avoid the enormous amount of calculation time, the probit model with a structured error component was introduced (Yai et al. 1997). We call this model the “structured probit model.” The coefficients are estimated by the simulation method, using the Geweke, Hajivassiliou, and Keane (GHK) recursive simulator (Train 2003). Although the structured probit model can reduce the computation time, it still takes much longer than the MNL model. Therefore, the MNL-based SUE is still popular in practical rail route
choice analysis in the Tokyo Metropolitan Area. For example, Kato et al. (2003) simulates the urban rail traffic flows in the Tokyo Metropolitan Area using the MNL model.

3.0 APPROACH

3.1. Characteristics of rail demand analysis

Hibino (2004) points out that the rail network in Tokyo has the following features that distinguish it from the road network: (i) the travel time is not affected considerably by traffic congestion; (ii) two or more links possibly run on the same rail line because there are sometimes several rail services, including the express service; (iii) two or more rail services going to different destinations possibly run on the same link; (iv) passengers can change rail services at the stations they pass during their journey; and (v) the transfer time at the station depends on the rail service. These features are considered in our comparative analysis. First, we do not consider the vehicle-to-vehicle congestion but incorporate the in-vehicle congestion as shown later. Next, with regard to (ii) and (iii), the route commonality is explicitly considered by using a structured probit model. (iv) and (v) are also incorporated by using the transportation network that includes the transfer links as well as by introducing the transfer time into the generalized cost function.

Additionally, this paper highlights the following three factors that are required in urban rail demand analysis, particularly in megacities: the in-vehicle congestion, stochastic route choice, and route choice set. This paper examines the impacts of these three factors on the performance of travel demand methods. First, it is often observed that rail commuters suffer from serious in-vehicle congestion during the morning peak hours. As the in-vehicle congestion varies among rail routes, a rail user
can choose a route by considering not only the travel time/cost but also the in-vehicle congestion. Thus, our comparative analysis includes the equilibrium-based route assignment method and compares its performance with the non-equilibrium-based method. Second, the urban rail network in megacities is so complicated that rail users could find it difficult to understand the network well. If it is assumed that users have incomplete information, the stochastic approach would be more suitable than the deterministic approach. This study then compares the performance of the deterministic assignment methods and stochastic assignment methods. Third, the urban rail network in megacities is so dense that it enables rail users to choose between various routes. When using the probabilistic-choice-based technique for demand analysis, we should define the individual choice set. We usually generate a choice set under the practical generation rule. However, this rule may bias the estimation results. Thus, our study compares the different choice-set generating rules in the practical context.

3.2. Comparison of traffic assignment methods

In this paper, the following six traffic assignment methods are compared: the MNL model, SMNP model, UE model, logit-based SUE model, probit-based SUE model, and AON assignment model.

3.2.1 MNL

MNL is a discrete-choice type model, in which a consumer chooses an option discretely by maximizing his/her utility with the random factor (Ben-Akiva and Lerman 1985). The conditional indirect utility function of a route from one origin to the destination is formulated as follows:

\[ U_{ij,r} = V_{ij,r} + \epsilon_{ij,r} \]

\[ = \theta_C \cdot GC_{ij,r} + \epsilon_{ij,r}, \quad \text{(1)} \]
where $V_{ijr}$ is the universal component of the indirect utility when the $r$th route is chosen from zone $i$ to $j$, $\epsilon_{ijr}$ is the error component of the utility following the independently and identically distributed (iid) Gumbel, $GC_{ijr}$ is the generalized cost including the travel time and travel cost of the $r$th route from zone $i$ to $j$, and $\theta_c$ is an unknown parameter. When the total volume of traffic flow from zone $i$ to $j$ is given as $Q_{ij}$, the expected volume of traffic flow $q_{ijr}$ choosing the $r$th route from zone $i$ to $j$ is expressed as

$$E[q_{ijr}] = Q_{ij} \cdot p_{ijr}$$

$$= Q_{ij} \cdot \frac{\exp(\lambda V_{ijr})}{\sum_{r' \in R_j} \exp(\lambda V_{ijr'})},$$

(2)

where $p_{ijr}$ is the probability of choosing the $r$th route from zone $i$ to $j$ and $\lambda$ is a scale parameter corresponding to the Gumbel distribution with $\lambda^2 = \pi^2 / 6\sigma^2$ ($\sigma^2$ is the variance of the Gumbel distribution). In the parameter estimation, $\lambda$ is assumed to be 1 because $\lambda$ cannot be estimated independently. The parameters are estimated by likelihood maximization estimation.

### 3.2.2 SMNP

SMNP was originally proposed by Yai et al. (1997). This model is one of the probit models with a simplified structure of error components with which the commonality of the routes can be taken into account. The model is formulated as follows. First, the additive function of a conditional indirect utility function consisting of a systematic component and an error component $\epsilon_{ijr}$ is as shown in eq. (1). Second, the error component is assumed to be divided into the following two parts: the first part $\eta_{ijr}$ is the systematic error component that is influenced by the length of the overlapping sections in the two routes; and the second part $\eta_{ijr}$ is the white noise that follows the...
normal distribution with mean 0 and variance $\sigma_{0ij}^2$ that is independent among the routes. 

Third, the variance of the systematic error is assumed to be in proportion to the corresponding route’s length, whereas the covariance of the systematic error is in proportion to the length of overlapping sections in the corresponding pair of routes. Then, the variance matrix of $\eta_{ij}^{-1}$ is shown as

$$\text{cov}(\eta_{ij}^{-1}, \eta_{ij}^{-1}) = d_{ij,q} \sigma^2,$$ (3)

where $d_{ij,q}$ denotes the length of the overlapping sections of a pair of the $r$th route and the $q$th route from $i$ to $j$, and $\sigma^2$ represents the variance per unit length. Fourth, the variances of all the routes in the same Origin-Destination (O-D) pair are assumed to be identical. Then, the variance matrix of the error component in the indirect utility function with respect to a specific route is shown as

$$\sum_{q} \left( d_{ij} \sigma^2 + \sigma_0^2 \right) \begin{pmatrix} 1 & \hat{w}_{ij,12} & \cdots & \hat{w}_{ij,1R} \\ \hat{w}_{ij,12} & 1 & \cdots & \hat{w}_{ij,2R} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{w}_{ij,1R} & \hat{w}_{ij,2R} & \cdots & 1 \end{pmatrix},$$ (4)

where $d_{ij}$ represents the sum of the lengths of all the routes that connect $i$ and $j$, $\sigma_0^2$ represents the variance, $\xi$ represents the parameter with respect to the covariance, and $w_{ij,q}$ is defined as follows:

$$w_{ij,q} = \begin{cases} 1 & \text{(if } q = r) \\ d_{ij,q}/d_{ij} & \text{(if } q \neq r) \end{cases}.$$ (5)

As $\left( d_{ij} \sigma^2 + \sigma_0^2 \right)$ cannot be estimated independently, the unknown parameter with respect to the variance of the error component reduces only $\xi$. Since the probability of choosing a route cannot be derived in the closed form in the probit model, the expected likelihood can be maximized using the following simulated probability function to estimate the coefficients and parameters:

$$p_{ij} = \int_{-\infty}^{\infty} \phi(x) dx,$$
where \( B = \left\{ \mathbf{v} \mid \mathbf{v}_r + \mathbf{v}_{r'} > \mathbf{v}_r' + \mathbf{v}_{r'}', \quad \forall r' \neq r \right\} \)

We apply the GHK recursive simulator (Train 2003) to the parameter estimation by following Yai et al. (1997).

3.2.3 UE

The deterministic UE assignment is one of the traffic assignment techniques in which the transportation system is assumed to fall into stable equilibrium when all users maximize their individual utility, including the congestion in choosing their routes (Sheffi 1985). The conditions of the UE are formulated as follows:

\[
q_{r,ij}^{*} \left(GC_{r,ij}^{*} - GC_{ij}^{*}\right) = 0, \quad (6)
\]

\[
GC_{r,ij}^{*} - GC_{ij}^{*} \geq 0, \quad (7)
\]

\[
q_{r,ij}^{*} \geq 0, \quad (8)
\]

where \( q_{r,ij} \) is the volume of the traffic flow of the \( r \) th route from \( i \) to \( j \), \( GC_{r,ij} \) is the generalized cost of the \( r \) th route, and \( GC_{ij} \) is the generalized cost from \( i \) to \( j \) in equilibrium. The symbol “**” denotes the equilibrium status. The relationship between (1) the traffic volume of a route and that of an O-D pair, (2) the traffic volume of a link and that of a route, and (3) the generalized cost of a route and that of a link are respectively shown as

\[
\sum_{r \in R_i} q_{r,ij} = Q_{ij}, \quad (9)
\]

\[
X_{l} = \sum_{q} \sum_{r \in R_{l}} q_{r,ij} \delta_{q,r,l}, \quad (10)
\]

\[
GC_{l} = \sum_{q} \sum_{r \in R_{l}} GC_{r,ij} \delta_{q,r,l}, \quad (11)
\]

where \( X_{l} \) is the traffic volume of link \( l \), \( \delta_{q,r,l} \) is equal to 1 if link \( l \) is on route \( r \) (otherwise it is 0), and \( GC_{l} \) is the generalized cost of link \( l \). The generalized cost of the link is
dependent upon the traffic flow of link $x_i$. The abovementioned conditions of the user equilibrium are theoretically equal to the optimization problem,

$$
\min_{x_i} \sum_{\omega} \int_{0}^{x_i} GC_i(\omega) d\omega,
$$

subject to eqs. (9) to (11).

This optimization problem is solved by the Frank-Wolfe method (Sheffi 1985). The UE traffic pattern arises if each traveler has perfect knowledge about the network conditions and all travelers have identical perceptions of generalized cost.

### 3.2.4 SUE

In reality, travelers select routes to improve their perceived generalized costs rather than their actual generalized costs, which are not perfectly known to them. The route choice principle for this case with the perception variations is often called the SUE. At the SUE, the perceived generalized costs on all used routes are equal and less than or equal to the perceived generalized costs on any unused route for each O-D pair. The perceived generalized cost is formulated in the same manner as eq. (1), in which the perceived generalized cost is converted into utility with the error component. Eqs. (9) to (11) should also be satisfied at the SUE. When the error component is assumed to follow the Gumbel distribution, it is known as the logit-based SUE (Dial 1971). The expected route flow of the logit-based SUE is expressed as eq. (2). When the error component is assumed to follow a normal distribution, it is known as the probit-based SUE (Daganzo and Sheffi 1977). In this paper, we use the probit-based SUE with the structured error component, which is the same as SMNP. We apply the method of successive averages both to the logit-based SUE and the probit-based SUE by following the method proposed by Fisk (1980).
3.2.5 AON assignment

AON assignment is a traffic assignment method in which all the traffic flows from an origin to a destination are simply allocated to the route with the lowest cost among the alternative routes. This technique does not take into account the change in the generalized cost due to traffic congestion or in-vehicle congestion. The traffic volume assigned by this method is shown as

\[ q_{ij,r} = Q_{ij} \quad \text{if} \quad GC_{ij,r} = \min\{GC_{ij,1}, GC_{ij,2}, \ldots\} \]

\[ = 0 \quad \text{otherwise}. \]  

(13a)

(13b)

4.0 EMPIRICAL ANALYSIS

4.1 Data used for the empirical analysis

The data used for the empirical analysis is the Tokyo Metropolitan Travel Census, 2000. This survey was conducted by a joint research team of the Ministry of Land, Infrastructure and Transport, Japan, and the Institute of Transport Policy Studies, Japan. The survey comprises three surveys: the first is a paper-based interview survey on the home-to-work and home-to-school travel of passengers with seasonal tickets; the second is a paper-based questionnaire survey on the travels of passengers without seasonal tickets; and the third is a paper-based interview survey on the service of rail operators. The passenger travel surveys were conducted in October 2000. The passenger travel data include the records of rail journeys with an origin and destination, travel purpose, travel route, and the beginning and end timings, along with sociodemographic data. The data were obtained from the Kanto region, which encompasses seven prefectures: Tokyo, Kanagawa, Saitama, Chiba, Ibaraki, Tochigi, and Gunma.
To estimate the unknown parameters in the models, a sample travel dataset was constructed through the following steps. First, we categorized the observed travels into the following four types: home-to-work, home-to-school, private, and business. Second, we chose travelers with one or more alternative routes. This eliminates travelers having very short travel times. Finally, we obtained 1,390 samples for home-to-work travel, 1,191 samples for home-to-school travel, 644 samples for private travel, and 462 samples for business travel. A rail network in the Tokyo Metropolitan Area is shown in Figure 1.

![Figure 1 is inserted here.]

We then prepared the level-of-service data for the selected samples. To do so, we constructed a rail network including the 1,877 zones, 4,850 nodes, and 9,796 links in the Tokyo Metropolitan Area. The network includes the links for the rail line, transfers at the rail stations, and access/egress travel to/from the rail stations; it also includes the nodes representing the origin/destination, platforms, and gates at the rail stations. When a rail line has more tracks in addition to the local service track, we set additional links to the local-service-track link. We also set transfer links connecting all the pairs of all the platforms at the stations. We set three to four access/egress links from an origin/destination zone to the adjacent stations considering the road and bus service networks around the zones. Then, we constructed the dataset including the travel time, transfer time, access/egress travel time, travel cost, and in-vehicle congestion rate for each link.
4.2. Parameter estimation of MNL and SMNP

For the parameter estimation of MNL and SMNP, the generalized cost function of a route is specified as the linear function shown below:

\[
GC_{ij,r}^a = C_{ij,r} + \frac{1}{a^{r}} \sum_{k \in C} a^{r} X_{k,ij,r} \]

\[
= C_{ij,r} + a^{r}_1 \theta_0 \cdot T_{ij,r} + a^{r}_2 \theta_0 \cdot T_{2ij,r} + a^{r}_3 \theta_0 \cdot T_{3ij,r} + a^{r}_4 \theta_0 \cdot Cong_{ij,r}, \tag{14}
\]

where \( T_{ij,r} \) is the access/egress travel time of the \( r \)th route from zone \( i \) to \( j \) for travel purpose \( a \), \( T_{2ij,r} \) is the rail-ride travel time, \( T_{3ij,r} \) is the transfer time at the station, \( C_{ij,r} \) is the travel cost, \( Cong_{ij,r} \) is the in-vehicle congestion, and \( a^{r}_m \) is the coefficient of the corresponding variables. The access/egress travel time is defined as the sum of the access travel time from the origin to the starting station and egress travel time from the final station to the destination. The transfer time includes the waiting time and walking time at the station where a passenger changes from one train to another. The in-vehicle congestion is defined as

\[
Cong_{ij,r} = \sum_{l \in L_{ij}} a^{r}_2 \cdot T_{2lj,r}, \tag{15}
\]

where \( a \) represents the congestion rate of a rail link \( l \) and \( T_{2lj,r} \) denotes the rail-ride travel time of rail link \( l \). The congestion rate is defined as

\[
\frac{\sum_{r} \sum_{a} \sum_{l} q_{ij,l,r}^{a}}{cap_{l}}, \tag{16}
\]

where \( q_{ij,l,r}^{a} \) represents the link traffic flow of link \( l \) of the \( r \)th route from zone \( i \) to \( j \) for travel purpose \( a \) and \( cap_{l} \) is the traffic flow capacity of link \( l \).

Then, we estimate the coefficients of MNL and SMNP using the empirical data shown earlier. GAUSS Ver.6.0 (Aptech System Inc.) is used for parameter estimations. The estimated results are shown in Table 1.
This shows that the models are well estimated from a statistical viewpoint. The results of the tests for most of the coefficients indicate that they are highly significant. The likelihood ratios are also high enough. The signs of all coefficients are also reasonable. The value of travel time savings of home-to-work, home-to-school, private, and business, estimated using MNL, are 23.0, 7.1, 8.5, and 11.7 yen/minute respectively, while the values of travel time savings of home-to-work, home-to-school, private, and business, estimated using SMNP, are 35.9, 11.5, 16.1, and 36.4 yen/minute respectively. Note that the average wage rate in Japan is 37.6 yen/minute as of 2000.

4.3. Definition of parameters in UE, SUE, and AON

We define the link performance functions for the UE and SUE assignment analyses. The link performance functions are expressed as follows:

Access/egress link: \( GC^n_i = C_i + \theta^n_i / \theta^n_i T_{ij} \), (17)

Rail link: \( GC^n_i = C_i + \theta^n_i / \theta^n_i T_{ij} + \rho^n_i / \theta^n_i Cong_{ij} \), (18)

Transfer link: \( GC^n_i = C_i + \theta^n_i / \theta^n_i T_{ij} \), (19)

where \( T_{ij} \) is the travel time of link \( i \) (\( n = 1, 2, \) and 3) and \( C_i \) is the travel cost of link \( i \).

The estimated results shown in Table 1 are used for the coefficients in the link performance functions. When using AON, we delete the in-vehicle congestion term from eq. (18) for the rail link performance function.
4.4. Methods to generate the route choice set

For the demand analysis with the MNL model, SMNP model, or SUE assignment, we must define the route choice set for all O-D pairs. Although the definition of route choice sets may significantly impact the traffic assignment results, it has not been well discussed in the context of the empirical rail route assignment problem in Tokyo. On the basis of the method currently used in practical demand analysis, the following four methods are examined to generate the route choice set in a given O-D pair: Methods 1, 2, 3, and 4. Two factors are considered differently among the four methods. The first factor is the variables used as the criteria for identifying the routes: generalized cost or travel time. The generalized cost is used in Methods 1 and 3, while the travel time is used in Methods 2 and 4. Note that the generalized cost is defined as the route-based performance consisting of the link performance functions shown in eq. (17), (18), or (19). The second factor is a heuristic search for the routes. Heuristic search refers to the method wherein an analyst specifies one route or multiple routes on the basis of his/her experiences after observing the travelers’ route choice and then adds it/them to the choice set of a given O-D pair. Methods 1 and 2 do not include heuristic search, while Methods 3 and 4 include it. The characteristics of the four methods are summarized in Table 2.

[Table 2 is inserted here.]

4.4.1 Method 1

Step 1-1: A route with the lowest generalized cost is sought in a given O-D pair.

Step 1-2: On the one hand, the three stations are specified by identifying the access links with the lowest, second lowest, and third lowest generalized cost respectively from the origin zone to the origin stations. On the other hand, the
three stations are specified by searching for the egress links with the lowest, second lowest, and third lowest generalized cost respectively from the destination stations to the destination zone. Then, the rail routes between the specific pairs of the specified origin stations and the specified destination stations are identified. A maximum of 9 routes are listed.

Step 1-3: Synthesize the route searched for in Step 1-1 and the routes searched for in Step 1-2 into a route list. If the route searched for in Step 1-1 includes the routes searched for in Step 1-2, it is eliminated from the list. A maximum of 10 routes are used in the final route list as the choice set of the corresponding O-D pair.

4.4.2 Method 2

Step 2-1: A route with the shortest travel time is sought in a given O-D pair.

Step 2-2: On the one hand, the three stations are specified by searching for the access links with the shortest, second shortest, and third shortest travel time respectively from the origin zone to the origin stations. On the other hand, the three stations are specified by searching for the egress links with the shortest, second shortest, and third shortest travel time respectively from the destination stations to the destination zone. Then, the rail routes between the specific pairs of the specified origin stations and the specified destination stations are searched for. A maximum of 9 routes are listed.

Step 2-3: Synthesize the route searched for in Step 2-1 and the routes searched for in Step 2-2 into a route list. If the route searched for in Step 2-1 includes those searched for in Step 2-2, it is eliminated from the list. Finally, a maximum of 10 routes are used in the final route list as the choice set of the corresponding O-D pair.
4.4.3 Method 3

Step 3-1: A route with the lowest generalized cost is searched for in a given O-D pair.

Step 3-2: On the one hand, the three stations are specified by searching for the access links with the lowest, second lowest, and third lowest generalized cost respectively from the origin zone to the origin stations. On the other hand, the three stations are specified by searching for the egress links with the lowest, second lowest, and third lowest generalized cost respectively from the destination stations to the destination zone. Then, the rail routes between the specific pairs of the specified origin stations and the specified destination stations are sought. A maximum of 9 routes are listed.

Step 3-3: In addition to Steps 3-1 and 3-2, the analyst heuristically specifies a maximum of 6 rail routes on the basis of the observed choices of rail users for a given origin zone.

Step 3-4: Synthesize the routes searched for in Steps 3-1, 3-2, and 3-3 into a route list. If the route searched for in Step 3-1 includes the routes searched for in Step 3-2 or Step 3-3, it is eliminated from the list. If the routes searched for in Step 3-2 include the routes searched for in Step 3-3, they are eliminated as well. A maximum of 16 routes are used in the final route list as the choice set of the corresponding O-D pair.

4.4.4 Method 4

Step 4-1: A route with the shortest travel time is searched for in a given O-D pair.

Step 4-2: On the one hand, the three stations are specified by searching for the access links with the shortest, second shortest, and third shortest travel time respectively from the origin zone to the origin stations. On the other hand, the three stations are specified by searching for the egress links with the shortest, second shortest, and third shortest travel time respectively from the
destination stations to the destination zone. Then, the rail routes between the specific pairs of the specified origin stations and the specified destination stations are sought. A maximum of 9 routes are listed.

Step 4-3: In addition to Steps 4-1 and 4-2, the analyst heuristically specifies several rail routes on the basis of the observed choices of rail users for a given origin zone.

Step 4-4: Synthesize the routes searched for in Steps 4-1, 4-2, and 4-3 into a route list. If the route searched for in Step 4-1 includes the routes searched for in Step 4-2 or Step 4-3, it is eliminated from the list. If the routes searched for in Step 4-2 include the routes searched for in Step 4-3, they are eliminated as well. A maximum of 16 routes are used in the final route list as the choice set of the corresponding O-D pair.

### 5.0 ANALYSIS RESULTS

#### 5.1. Calculations of traffic flows with the models

O-D matrixes of urban rail users are prepared for traffic assignment in the Tokyo Metropolitan Area. First, the all-mode O-D matrixes are estimated by type of travel. For the home-to-work and home-to-school travels, O-D matrixes are estimated with the data of the National Census in 2000 and the Person Trip (PT) Survey conducted in the Tokyo Metropolitan Area in 1998. Note that although the National Census covers the population all over the nation, it collects data of only home-to-work and home-to-school travels. On the other hand, although the PT Survey is a sample-based survey conducted in the Tokyo Metropolitan Area, it collects data of all types of travels. Note also that only the city- or village-based O-D matrix is available in the National Census, while the zone-based O-D matrix is available in the PT Survey. Thus,
we divide the O-D matrix from the National Census into the detailed O-D matrix so as to match our zoning system by using the O-D pattern of the PT Survey. Next, for private and business travels, O-D matrixes are estimated with the data of the PT Survey conducted in the Tokyo Metropolitan Area in 1998. The O-D matrix from the PT survey in 1998 is revised to the O-D matrix in 2000 by using the zone-based population. Next, the estimated all-mode O-D matrixes are split into rail-use O-D matrixes by type of travel. We use the MNL-based choice models that were used in the rail demand forecast for preparing the latest master plan of the urban rail network in the Tokyo Metropolitan Area (Morichi et al. 2001).

Then, we analyze the traffic flows in the network using the six methods, incorporating the four methods to generate the route choice set. In the analysis with the UE and SUE assignments, we calculate the traffic flows of the home-to-work travels and home-to-school travels using the multi-class assignment method. This reflects the fact that the most of the home-to-work and home-to-school travels are engaged in during the morning peak hours. Therefore, the home-to-work travel flows intersect with the home-to-school travel flows via in-vehicle congestion. We calculate the traffic flows of private and business travels with AON assignment under the assumption that they are free from in-vehicle congestion. This is because it is assumed that most of the private and business travels usually do not start during the peak hours.

5.2. Comparison of model fitness among traffic assignment methods

Table 3 shows a comparison of the computation time with the iteration times among the methods.

[Table 3 is inserted here.]
First, AON has the shortest computation time, followed by MNL with Method 2 and MNL with Method 1. Second, the stochastic assignment methods require a longer computation time than the deterministic assignment methods. MNL or SMNP require a longer computation time than AON, while logit-based or probit-based SUE requires a longer computation time than UE. This is because the stochastic methods entail computation for multiple routes, while the deterministic methods require computation for a single route. Third, the computation time of the probit-based method is about 2–3 times longer than that of the logit-based method. This is because the simulation of probability calculations requires a longer computation time in the probit model than in the logit model. Fourth, the methods using the equilibrium concept require a longer computation time than the non-equilibrium methods. UE requires a longer computation time than AON, while logit-based SUE and probit-based SUE require a longer computation time than MNL and SMNP respectively. Fifth, the computation time per iteration of Method 1 is longer than that of Method 2, while the computation time per iteration of Method 3 is almost equal to that of Method 4. Finally, SMNP with Method 4 completes the computation in about half an hour. Note that SMNP with Method 4 was used in the latest urban rail development plan in the Tokyo Metropolitan Area.

Next, Table 4 summarizes the model fitness of the traffic assignment methods. Figure 2 shows a comparison of the computed link flows with the observed link flows among the methods.

[Table 4 and Figure 2 are inserted here.]

First, the probit-based SUE with Method 3 has the best fitness, followed by the probit-based SUE with Method 4, while SMNP with Method 2 has the worst fitness and SMNP with Method 1 has the second worst fitness. Second, Methods 3 and 4 have a better fitness than Methods 1 and 2 in all types of traffic assignment methods. This
means that the heuristic method is more preferable to the non-heuristic method for selecting the routes in the choice set. Third, Method 1 has a better fitness than Method 2 for all types of methods. This means that it is better to use the generalized cost when identifying the routes to generate the choice set than to use the travel time if the non-heuristic method is used for defining the route choice set. However, note that the fitness of Method 3 is not always better than that of Method 4. Fourth, surprisingly, AON has a better fitness than MNL with Method 1 or 2 and SMNP with Method 1 or 2. This may reflect the bias in defining the route choice set by Method 1 or 2. Fifth, the fitness of UE is better than that of AON. This indicates that it is beneficial to take the equilibrium into account in rail route choice. Sixth, the fitness of SMNP with Method 1 or 2 is worse than that of MNL with Method 1 or 2 respectively, whereas the fitness of SMNP with Method 3 or 4 is better than that of MNL with Method 3 or 4 respectively. Note that the fitness of the probit-based SUE is better than that of the logit-based SUE, except with Method 2. This means that the estimation results can be improved to some extent by considering the route commonality. Finally, SMNP with Method 4 has the third best fitness among the traffic assignment methods. Note again that SMNP with Method 4 was used in the latest urban rail development plan in the Tokyo Metropolitan Area.

5.3. Comparison of link flows in specific routes among the traffic assignment methods

Figures 3 and 4 show the two cases in which the link flows computed by the four traffic assignment methods are compared in the specific rail lines.

[Figure 3 and Figure 4 are inserted here.]
Figure 3 shows the link flows of two rail lines connecting Omiya with Akabane, and Figure 4 shows the link flows of two rail lines connecting Mitaka with Ochanomizu. Note that both the line services run through different stations, although their start and end stations are the same. Note also that they are all operated by the same rail company. Figures 5 and 6 show the rail network covering the two rail lines connecting Omiya with Akabane and Mitaka with Ochanomizu.

[Figure 5 and Figure 6 are inserted here.]

The four methods compared in these cases are AON, UE, MNL with Method 4, and SMNP with Method 4.

First, the link flows computed by AON are similar to the link flows computed by UE, while the link flows computed by MNL are similar to those computed by SMNP. This is because AON and UE assign all the traffic in a given O-D demand to the shortest path. AON and UE underestimate the link flows in a longer travel time route, while they overestimate the link flows in a shorter travel time route. Second, the error in computed link flow in AON is larger than that in UE. This is because the UE incorporates the equilibrium but AON does not. Third, Figure 2 shows that the error of SMNP is larger than that of MNL. This is probably because the parameters of SMNP are more sensitive than those of MNL.

5.4. Discussion

First, incorporating the route commonality improves the accuracy to some extent but increases the computation time. The methods incorporating the route commonality, including SMNP and probit-based SUE, have better accuracy than the methods that do not consider the route commonality when Method 3 or 4 is used for
generating the route choice set. However, the methods incorporating the route commonality require a 2–3 times longer computation time than those that do not. Second, the equilibrium-based methods have better accuracy than the non-equilibrium-based methods; however, they require a much longer computation time. The fitness of UE is better than that of AON, but the computation time of UE is 10 times longer than that of AON. This means that the urban rail passengers consider the in-vehicle congestion when choosing the route in Tokyo. Third, the stochastic assignment methods do not always have better accuracy than the deterministic methods. One of the possible reasons for this is the existence of a bias in the methods to generate the route choice set. However, this should be examined further. Fourth, the heuristics in generating the choice set have a critical impact on accuracy. Although SMNP with Method 4, which was used in the latest rail development plan in Tokyo, has the third best fitness among the methods, SMNP with Method 2 has the worst fitness among them. This may mean that the high accuracy of SMNP with Method 4 is much dependent on the heuristic method to generate the route choice set. Finally, it should be noted that it is important to consider the context of application when determining the assignment method. On the one hand, the probit-based SUE is the best from the accuracy viewpoint. However, on the other hand, the probit-based SUE required a long computation time. Four or five hours of computation time is too long in practice. For example, the latest urban rail planning in the Tokyo Metropolitan Area in 2000 examined over 1000 cases in a period of six months (Morichi et al. 2001). If we were to use the probit-based SUE to examine 1000 case studies, it would take more than five months only to predict the future traffic demand, which is not realistic in the context of the Tokyo Metropolitan Area. Thus, it may be necessary for the analysts to choose the second- or third-best methods in terms of accuracy under the constraints of computation time in the practical planning context.
6.0 CONCLUSIONS

This paper compares the performance of six traffic assignment methods with the same empirical dataset of route choice. MNL, SMNP, UE assignment, logit-based SUE assignment, probit-based SUE assignment, and AON assignment are applied to the comparative empirical analysis. The revealed preference data of urban rail route choice in the Tokyo Metropolitan Area are used for the empirical case analysis. The empirical case analysis shows that the probit-based SUE simulates link traffic flows with the highest accuracy. However, the probit-based SUE requires almost seven hours. It also shows that the heuristics to generate the choice set influence accuracy, while incorporating the route commonality and the in-vehicle congestion significantly improves the accuracy.

In the latest urban rail planning in the Tokyo Metropolitan Area, the planning process requires the computation time of traffic demand analysis to be around an hour. Note that the traffic demand analysis includes not only the rail route choice analysis but also the modal choice. Therefore, the required computation time for the network assignment process is less than an hour. This makes it difficult for transport demand analysts and urban rail planners to select an analysis method for urban route choice analysis; this is because the computation time using an assignment method with high accuracy is longer than the required time, as shown earlier. There are three potential solutions to the abovementioned problems with regard to future rail planning in Tokyo. The first way is simply to accept the increase in the computation time. If more time is given to the travel demand analysis in the planning process, we could increase the computation time. However, on the whole, this makes the planning process longer. The second way is to make the required accuracy lower than that obtained with the probit-based SUE. The simplification of the zoning and/or network can also contribute to the decrease in computation time, although the accuracy may be reduced. However, this modeling would not be state-of-the-art. The third way is to introduce a new method. For
example, the C-logit model (Cascetta et al. 1997) may be an alternative to SMNP. As the choice probability in the C-logit is the same as that of MNL (except the commonality factor in the utility function), the computation time with the C-logit SUE is expected to be almost the same as that with the logit-based SUE. However, the logit-based SUE with Method 3 or 4 still requires more than an hour for a unit computation.

Finally, we can be certain that the performance of computers will improve rapidly. On the other hand, more complicated and sophisticated analyses, which require rougher computation with richer databases, may also be expected by policy makers, planners, and engineers. State-of-the-art analysis could contribute to better public transit planning and a smoother consensus-building process. Thus, there is still a need for developing a more efficient assignment method for the public transit system in the future.

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References


Dial RB (1971) A probabilistic multi-path traffic assignment model which obviates the need for path enumeration. Transp Res 5:83–111


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Table 1 Estimation results of urban rail route choice models with MNL and SMNP

<table>
<thead>
<tr>
<th>Variables</th>
<th>Units</th>
<th>MNL</th>
<th></th>
<th>MNL</th>
<th></th>
<th>SMNP</th>
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<th>SMNP</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Home-to-work</td>
<td>Home-to-school</td>
<td>Private</td>
<td>Business</td>
<td>Home-to-work</td>
<td>Home-to-school</td>
<td>Private</td>
<td>Business</td>
</tr>
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<td>Rail-ride travel time</td>
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<td>–0.0902 (–9.9)</td>
<td>–0.057 (–5.0)</td>
<td>–0.0337 (–3.4)</td>
<td>–0.1145 (–4.7)</td>
<td>–0.1637 (–5.9)</td>
<td>–0.135 (–2.9)</td>
<td>–0.1148 (–1.9)</td>
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<td>Transfer time</td>
<td>Minutes</td>
<td>–0.1153 (–13.6)</td>
<td>–0.1335 (–13.6)</td>
<td>–0.1019 (–8.9)</td>
<td>–0.0854 (–6.6)</td>
<td>–0.1422 (–6.4)</td>
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<td>–0.1746 (–4.0)</td>
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<td>Access/egress travel time</td>
<td>Minutes</td>
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<td>–0.218 (–4.4)</td>
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<td>–0.00709 (–1.9)</td>
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<td>–</td>
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<td>Variance</td>
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<td>(–)</td>
<td>(–)</td>
<td>(–)</td>
<td>(–)</td>
<td>0.189 (1.9)</td>
<td>0.211 (2.7)</td>
<td>0.41 (1.5)</td>
<td>0.513 (1.0)</td>
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<td>462</td>
<td>1,390</td>
<td>1,191</td>
<td>644</td>
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Table 2 Four methods of generating the choice set in the rail route choice between an O-D pair

<table>
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<tr>
<th>Heuristic route search is included</th>
<th>Generalized travel cost is used for route search</th>
<th>Travel time is used for route search</th>
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<td>Heuristic route search is not included Method 1</td>
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<td>Method 3 Method 4</td>
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Table 3 Comparison of computation performance among traffic assignment methods

<table>
<thead>
<tr>
<th>Traffic assignment methods</th>
<th>Methods to define the choice set</th>
<th>Iteration times</th>
<th>Computation time (minutes)</th>
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<td>UE</td>
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<td>17</td>
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<tr>
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</tr>
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<td>Logit-based SUE Method 1</td>
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Table 4 Comparison of the fitness among traffic assignment methods

<table>
<thead>
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<th>$R^2$</th>
<th>Root mean square error</th>
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<td>33125</td>
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</table>
Figure 2
Figure 3
Chuo Line [Rapid Service]

Chuo Line, Sobu Line [Local Service]

Figure 4
Figure 5

Keihin-Tohoku Line

Figure 6

Chuo Line [Rapid Service]

Chuo Line, Sobu Line [Local Service]